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IDENTIFICATION OF THE CHARACTERISTICS OF SURFACE THERMAL INTERACTION BETWEEN MATERIALS AND GAS STREAMS

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The authors analyze the possible identification of the functional parameters in the energy balance equation on the disintegrating surface of a solid.

Mathematical modeling of processes of thermal interaction of disintegrating structural and heat shield materials with high enthalpy gas streams must be based, in general, on solving the coupled problems of unsteady heat and mass transfer. Problems of this class are formulated in the form of a single system of equations describing the whole complex of interconnected processes: the gas flow in the inviscid region; the heat and mass transfer in the high-temperature boundary layer in the presence of blowing and chemical reactions in multicomponent gas mixtures; and surface disintegration and heat transfer within the material.

Solution of the coupled heat- and mass-transfer problems in the full formulation is a complex problem, and one that is difficult to solve at present. Therefore, one must construct simplified mathematical models which describe approximately the complex processes under examination. And here one must include the basic factors influencing thermal interaction of the material with the gas stream [1, 2].

Approximate mathematical models usually contain a number of effective values of characteristics, each of which takes account of a certain set of individual phenomena and processes. Methods of parametric identification of inverse heat-transfer problems [3] have recently found widespread use in determining these characteristics.

In this paper we analyze the inverse problem of recovering the characteristics of surface thermal interaction of a disintegrating material with a high enthalpy gas stream. Here we assume that the heat-transfer process within the material is described by the homogeneous heat-conduction equation, one-dimensional in a space coordinate, with coefficients that are functions of temperature. In addition, it is assumed that the disintegration and removal of material occurs only in the gas phase. This corresponds to the mechanism of thermochemical disintegration of the surface of subliming materials. In the more general case, e.g., for composite materials, other factors [1, 2] must be accounted for.

With these assumptions the approximate mathematical model of the process of heat and mass transfer occurring in a certain time interval $(0, \tau_m]$ in the gas–solid system, allowing for disintegration of the material on the wetted surface, can be represented in the form of the following heat-conduction boundary problem:

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$$c(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right), \quad x \in (0,\ b(\tau)), \quad \tau \in (0,\ \tau_m], \tag{1}$$

$$b(\tau) = b(0) - \int_0^{\tau_m} G_w(\tau)/\rho(T)\, d\tau, \tag{2}$$

$$T(x,\ 0) = T_0(x), \quad x \in [0,\ b(0)], \tag{3}$$

$$-\lambda(T(0,\ \tau))\frac{\partial T(0,\ \tau)}{\partial x} = q_{in}(\tau), \quad \tau \in (0,\ \tau_m], \tag{4}$$

$$T_w = T(b(\tau),\ \tau), \quad -\lambda(T_w)\left(\frac{\partial T}{\partial x}\right)_w = q_\lambda(\tau), \quad \tau \in (0,\ \tau_m], \tag{5}$$

where the heat-flux density $q_\lambda(\tau)$ reaching the solid at the moving boundary $x = b(\tau)$ is determined from the energy balance equation on the disintegrating surface:

$$q_\lambda(\tau) = q_{w_0}(\tau) - \gamma G_w(\tau)(I_e - I_w) - G_w(\tau) Q_w(G_w) - \varepsilon(T_w)\sigma T_w^4. \tag{6}$$

To solve the system of equations (1)-(6) one must know all the characteristics of the process: $c(T)$, $\lambda(T)$, $G_w(\tau)$, $\rho(T)$, $T_0(x)$, $q_{in}(\tau)$, $q_{w_0}(\tau)$, γ, I_e, I_w, $Q_w(G_w)$, $\varepsilon(T_w)$, and the initial thickness $b(0)$.

With the available approximate numerical methods one can calculate, with sufficient accuracy for practical purposes, the heat flux supplied to the disintegrating surface, allowing for blowing of the gaseous disintegration products into the boundary layer, i.e., one can determine the heat-flux density

$$q_w(\tau) = q_{w_0}(\tau) - \gamma G_w(\tau)(I_e - I_w) \tag{7}$$

for a given value of the rate of formation of disintegration products $G_w(\tau)$ [4]. There are also experimental methods, and experimental–theoretical methods, based on solving the coefficients of inverse heat-conduction problems, which yield reliable data on the thermophysical characteristics of a material $c(T)$, $\lambda(T)$, $\rho(T)$ [5-7]. If these quantities are known, then one must identify the characteristics of the heat- and mass-transfer process on the disintegrating surface.

We shall assume that in the process of experimental investigations one has obtained the disintegration rate as a function of time $G(\tau)$, and we consider the problem of recovering the functions $\varepsilon(T_w)$ and $Q_w(G_w)$ describing the thermal interaction of the material with the gas stream from the results of measured temperature at internal points of the solid

$$T^{\exp}(X_i,\ \tau) = f_i(\tau), \quad X_i \in (0,\ b(\tau_m)), \quad i = \overline{1,\ N}. \tag{8}$$

One can construct an algorithm for solving the inverse problem thus posed in two phases: 1) from conditions (1)-(5) and (8) one recovers the thermal boundary conditions $q_\lambda(\tau)$ and $T_w(\tau)$ on the boundary with coordinate $x = b(\tau)$ by solving the inverse boundary problem (using the technique of [8], for example); and, 2) one identifies the unknown characteristics $\varepsilon(T_w)$ and $Q_w(G_w)$ from use of the algebraic heat-balance equation

$$q_\lambda(\tau) = q_w(\tau) - G_w(\tau) Q_w(G_w) - \varepsilon(T_w)\sigma T_w^4. \tag{9}$$

The second phase is the subject of the present analysis.

It is evident that the problem of recovering the two functions $\varepsilon(T_w)$ and $Q_w(G_w)$ from the single Eq. (9) does not have a unique solution. To avoid the nonuniqueness, one must bring in additional data. A natural choice for this additional information is to use the data of some set of thermophysical experiments conducted on identical specimens but in different thermal disintegration regimes, under the hypothesis that in the different experiments the unknown characteristics have the same relationships. Here it is necessary that the number of different regimes should be at least no less than the number of unknown characteristics. With this approach one can provide a number of algebraic equations of the type of Eq. (9) equal to the number of desired characteristics, or even an excess of experimental information.

Using the data of several experiments, we shall determine the unknown characteristics $\varepsilon(T_w)$ and $Q_w(G_w)$ from the condition of minimizing the functional

$$I = \sum_{i=1}^{M} \int_0^{\tau_{mi}} [q_{wi}(\tau) - q_{\lambda i}(\tau) - G_{wi}(\tau) Q_w(G_{wi}) - \varepsilon(T_{wi}) \sigma T_{wi}^4]^2 \, d\tau, \tag{10}$$

where $i = \overline{1, M}$ is the experiment number, and τ_{mi} is its duration.

We shall represent the regions for determining the functions $\varepsilon(T_w)$ and $Q_w(G_w)$ in the form of the intervals

$$\Omega_T = [T_{\min}, T_{\max}] \text{ and } \Omega_G = [G_{\min}, G_{\max}],$$

where

$$T_{\min} = \max_i (T_{wi})_{\min}; \quad T_{\max} = \min_i (T_{wi})_{\max};$$

$$G_{\min} = \max_i (G_{wi})_{\min}; \quad G_{\max} = \min_i (G_{wi})_{\max};$$

$(T_{wi})_{\max}$ is the maximum value of temperature; $(T_{wi})_{\min}$ is the minimum value of temperature; $(G_{wi})_{\max}$, maximum value of the rate of removal of material; and $(G_{wi})_{\min}$, minimum value of rate of removal of material in the i-th experiment.

We shall divide the two intervals into m_T and m_G sections, respectively, and obtain the mesh:

$$\omega_T = \{T_k = T_{\min} + k\Delta T, \quad k = \overline{0, m_T}\}, \quad \omega_G = \{G_k = G_{\min} + k\Delta G, \quad k = \overline{0, m_G}\}. \tag{11}$$

We represent the functions to be determined in the form of cubic B-splines [9]:

$$\varepsilon(T_w) = \sum_{k=-1}^{m_T+1} \varepsilon_k B_k(T_w), \tag{12}$$

$$Q_w(G_w) = \sum_{k=-1}^{m_G+1} Q_k B_k(G_w), \tag{13}$$

where

$$B_k(S) = B_0(S - k\Delta S - S_{\min}), \quad B_0(S) = \frac{1}{6\Delta S^3} [(S + 2\Delta S)_+^3 - 4(S + \Delta S)^3 + 6(S)_+^3 - 4(S - \Delta S)_+^3 + (S - 2\Delta S)_+^3],$$

and S is replaced by T_w or G_w, respectively.

Following parameterization we transfer from continuous-function space to real $(m_T + m_G + 6)$-dimensional space. After differentiating the functional (10) to be minimized with respect to the approximation parameters ε_k, $k = \overline{-1, m_T + 1}$ and Q_n, $n = \overline{-1, m_G + 1}$, we obtain the system of $(m_T + m_G + 6)$ linear equations:

$$\sum_{k=-1}^{m_T+1} \left(\sum_{i=1}^{M} \int_0^{\tau_{mi}} \sigma^2 T_{wi}^8 B_k(T_{wi}) \, B_l(T_{wi}) \, d\tau \right) \varepsilon_k + \sum_{n=-1}^{m_G+1} \left(\sum_{i=1}^{M} \int_0^{\tau_{mi}} G_{wi}(\tau) T_{wi}^4 B_n(G_{wi}) B_l(T_{wi}) \, d\tau \right) Q_n =$$

$$= \sum_{i=1}^{M} \int_0^{\tau_{mi}} (-q_{wi}(\tau) + q_{\lambda i}(\tau)) \sigma T_{wi}^4 B_l(T_{wi}) \, d\tau, \quad l = \overline{-1, m_T + 1},$$

$$\sum_{k=-1}^{m_T+1} \left(\sum_{i=1}^{M} \int_0^{\tau_{mi}} G_{wi}(\tau) \sigma T_{wi}^4 B_k(T_{wi}) B_m(G_{wi}) \, d\tau \right) \varepsilon_k + \sum_{n=-1}^{m_G+1} \left(\sum_{i=1}^{M} \int_0^{\tau_{mi}} G_{wi}^2(\tau) B_n(G_{wi}) B_m(G_{wi}) \, d\tau \right) Q_n \tag{14}$$

$$= \sum_{i=1}^{M} \int_0^{\tau_{mi}} (-q_{wi}(\tau) + q_{\lambda i}(\tau)) B_m(G_{wi}) \, G_{wi}^2(\tau) \, d\tau, \quad m = \overline{-1, m_T + 1}.$$

By solving the system (14) for ε_k, $k = \overline{-1, m_T + 1}$ and Q_n, $n = \overline{-1, m_G + 1}$, we obtain an approximate solution of the inverse problem being analyzed.

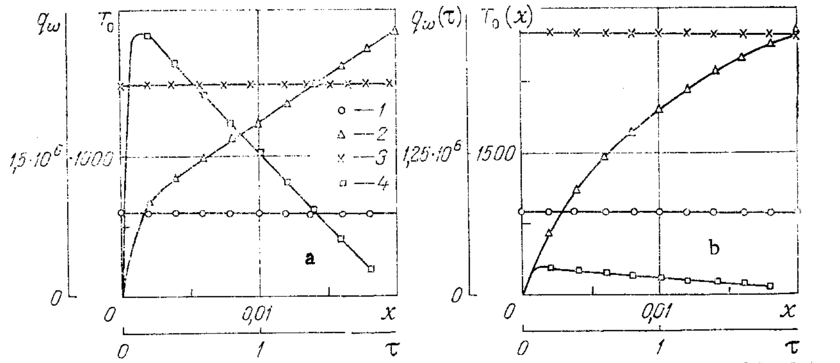

Fig. 1. Dependence of the heat-flux density $q_w(\tau)$ (W/m^2) and of the initial temperature distributions $T_0(x)$, °K, for the first (a) and second (b) groups of experiments: 1) T_0; 2) q_w (experiment 1); 3) T_0; 4) q_w (experiment 2).

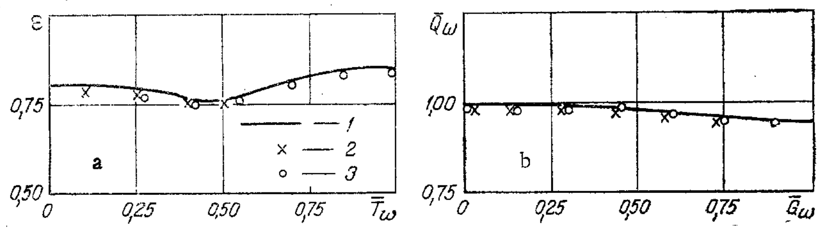

Fig. 2. Recovery of the integral emissivity $\varepsilon(\bar{T}_w)$ (a) and the thermal effect of surface transformations $\bar{Q}_w(\bar{G}_w)$ (b) without allowance for error: 1) the given dependence; 2) group of experiments I; 3) group of experiments II.

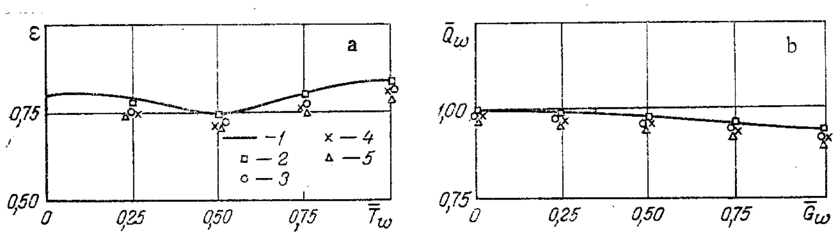

Fig. 3. Recovery of the integral emissivity $(\bar{T}_w)$ (a) and of the thermal effect of surface transformations $\bar{Q}_w(\bar{G}_w)$ (b) with allowance for error: 1) the given dependence; 2) recovery without allowance for error; given with errors: 3) $T_w(\tau)$; 4) $G_w(\tau)$; 5) $q_\lambda(\tau)$. Δ_{max} = 5%.

As was noted above, the original problem was incorrectly posed and, therefore, to avoid instability arising from the numerical solution it is desirable to use the principle of step-size regularization [3], which in this case comes down to reducing the number of parameters of the approximation.

On the basis of the suggested algorithm we developed a computer program with the help of which we conducted a numerical experiment. As the test material we chose graphite, whose thermophysical characteristics have been given in [1]. We modeled the heating and disintegration of a graphite specimen of thickness 0.02 m in one-sided interaction with an unsteady high-enthalpy air stream. The internal boundaries of the specimen were considered to be thermally insulated, and the initial temperature distribution to be constant.

We examined two groups of experiments: there were two experiments in each. Figure 1a shows the initial temperature distributions and the external heat flux densities for the first group of experiments, and Fig. 1b shows the same thing for the second group. All the experiments were of 5-sec duration.

The sequence of operations in the numerical experiment was as follows. From the given characteristics of the process [including $\varepsilon(T_w)$ and $Q_w(G_w)$] for each experiment we solved the direct problem of heat and mass transfer and calculated the temperature field and also the heat-flux density arriving at the body, $q_\lambda(\tau)$, and the rate of mass removal $G_w(\tau)$. Thereafter we solved the inverse problem of recovering the dependences $\varepsilon(T_w)$ and $Q_w(G_w)$. Here the approximating splines were constructed on the meshes, using three sections of approximation for each. Then we compared the given and the recovered values of the characteristics being analyzed.

Below we present results obtained in mathematical modeling of the two groups of experiments. Here as dimensionless coordinates we used the following quantities: $\bar{T}_w = T_w/T_{max}$, $\bar{Q}_w = Q_w/Q_{max}$, $\bar{G}_w = G_w/G_{max}$. The quality of recovery of the desired characteristics is shown at the intersection of regions of variation of the corresponding arguments achieved in each of the groups of experiments.

The results of the identification without allowing for errors arising in determining the relations $\varepsilon(T_w)$ and $Q_w(G_w)$ are shown in Fig. 2. It can be seen that the results practically coincide for the two groups of experiments. This confirms the nonformalized hypothesis made in this work that one can obtain a unique solution of the inverse problem considered.

Figure 3 shows the influence on the inverse-problem solution of errors in assigning each of the functions $G_w(\tau)$, $T_w(\tau)$, and $q_\lambda(\tau)$. The errors were modeled with the aid of a pseudorandom number sensor with a normal distribution law. The maximum relative error was Δ_{max} = 5%. The results presented are evidence of the rather high efficiency and numerical stability of the algorithm described.

NOTATION

T, temperature; c(T), volume heat capacity; $\lambda(T)$, thermal conductivity; x, coordinate; τ, time; τ_m, process duration; $q(\tau)$, specific heat flux; $b(\tau)$, test specimen thickness; γ, blowing parameter; I, gas stream enthalpy; $G_w(\tau)$, mass rate of material removal; $Q_w(G_w)$, thermal effect of surface processes; $\varepsilon(T)$, integral emissivity; σ, Stefan–Boltzmann constant; $f_i(\tau)$, measured temperatures. Subscripts: max, min, maximum and minimum values, respectively; w, characteristics at the disintegrating surface.

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